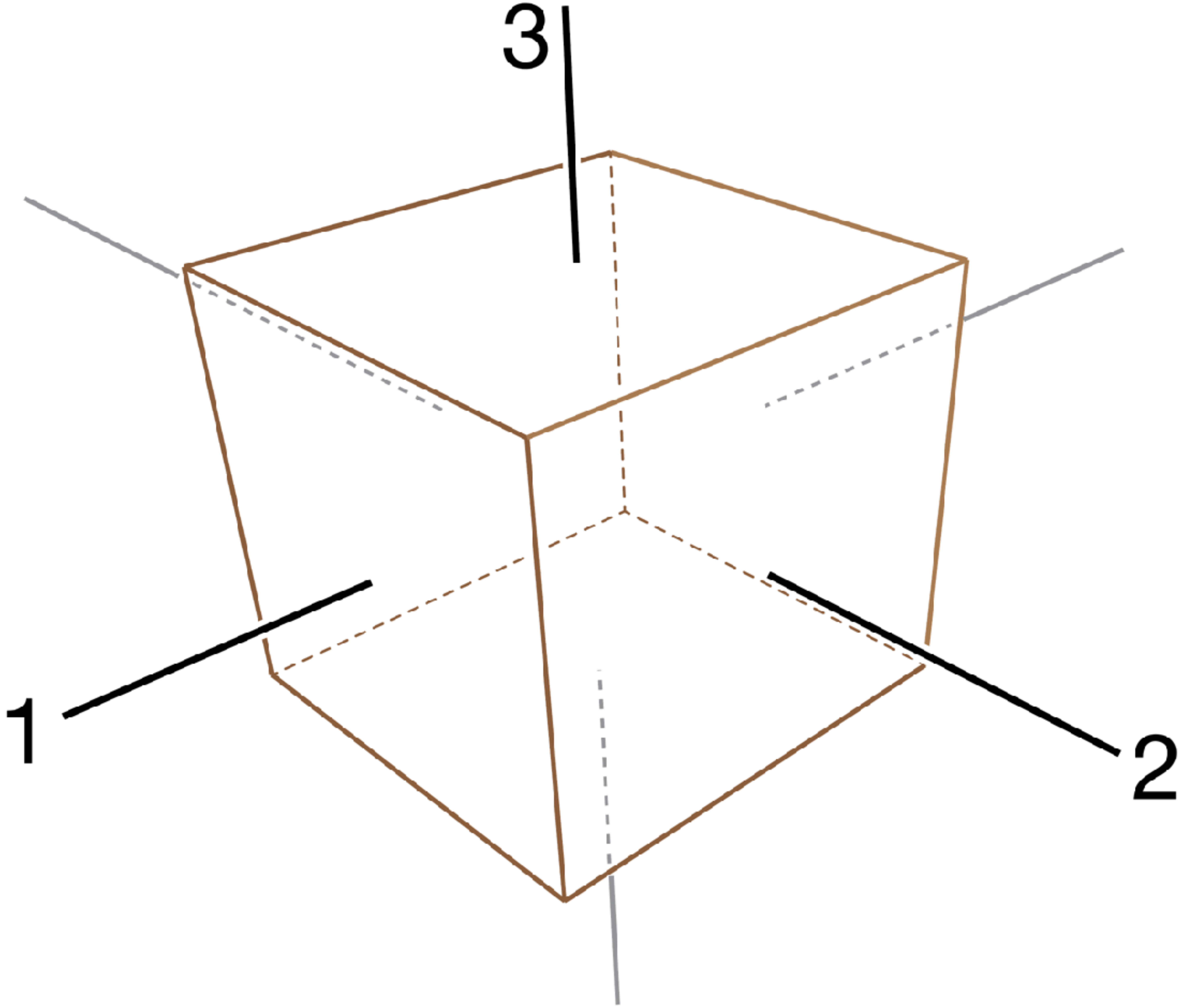


3
1
2

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

3x3 stress matrix

normal stresses

$$
\begin{matrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{matrix}
$$

shear stresses

$$\begin{matrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{matrix}$$

axis or diagonal of the matrix

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

conjugate shear stresses

axis or diagonal of the matrix

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

conjugate shear stresses

axis or diagonal of the matrix

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

conjugate shear stresses

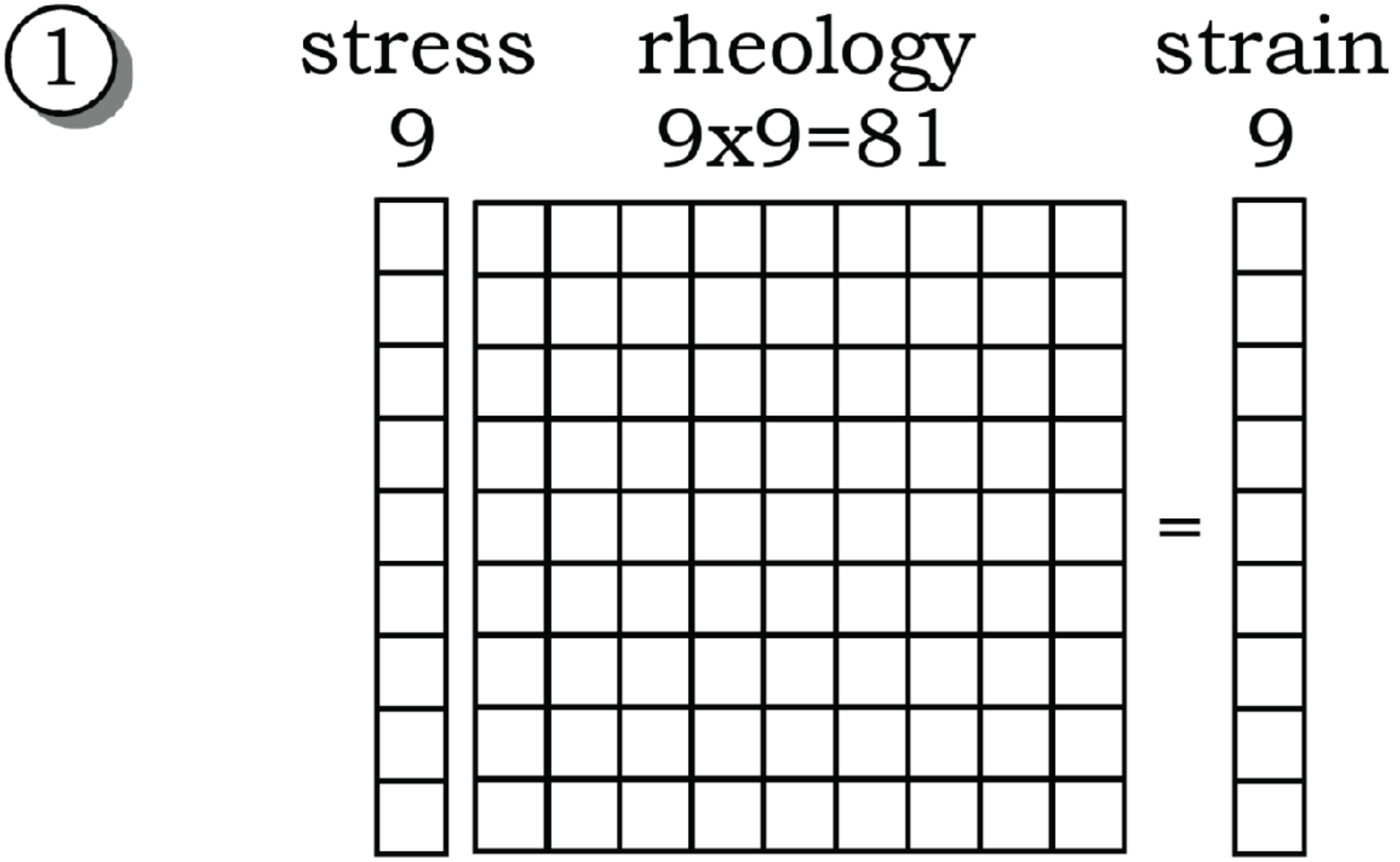
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

6 unique/independent stresses

If you apply a sufficient differential **stress** to a deformable mass, it will **strain**. The manner in which it strains is dependent on its material properties, which are expressed in its **rheology**.

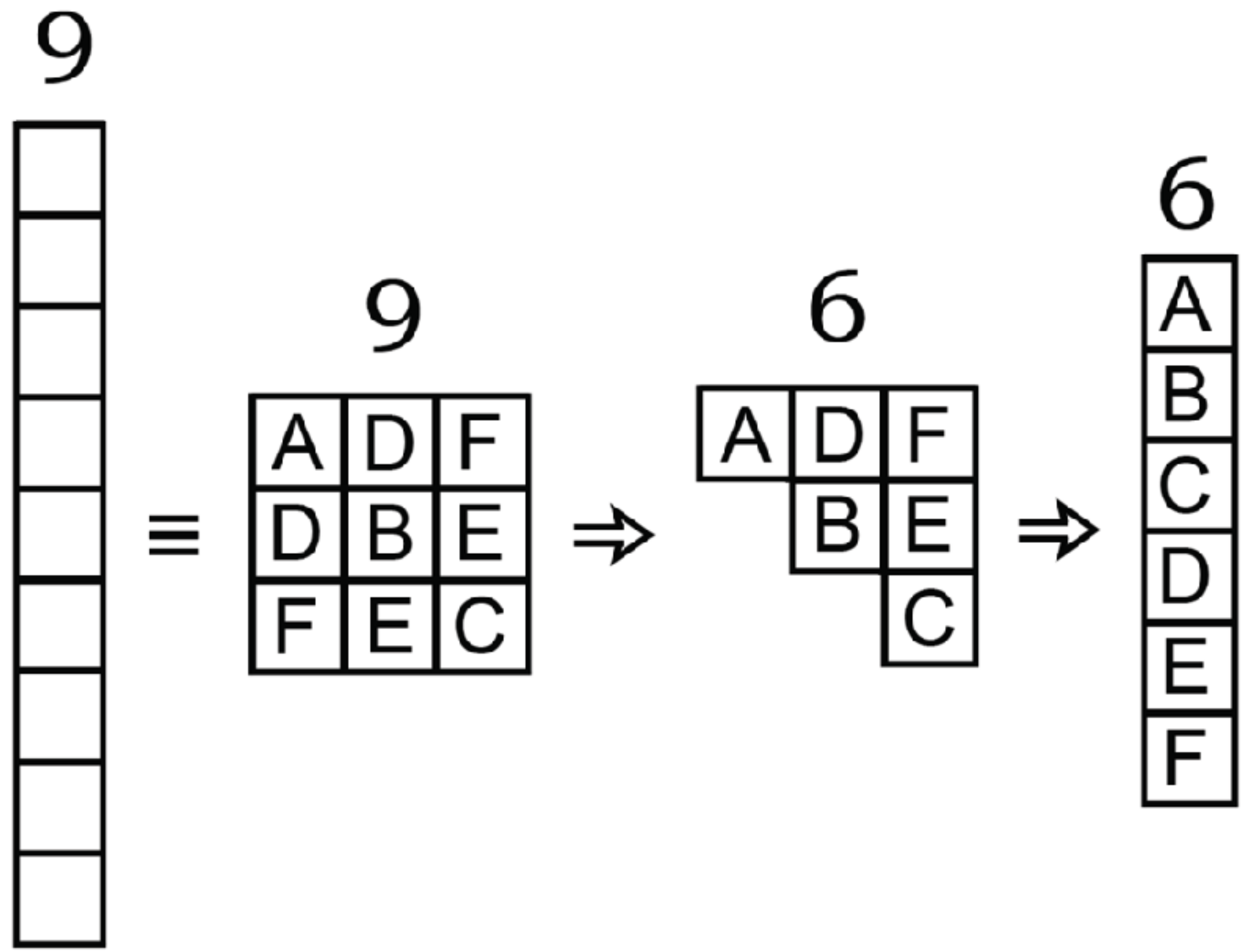
[**strain**] = [**rheology**] [**stress**]

Continuum mechanics provides the tools for the quantitative analysis of how stresses and strains are related to each other.



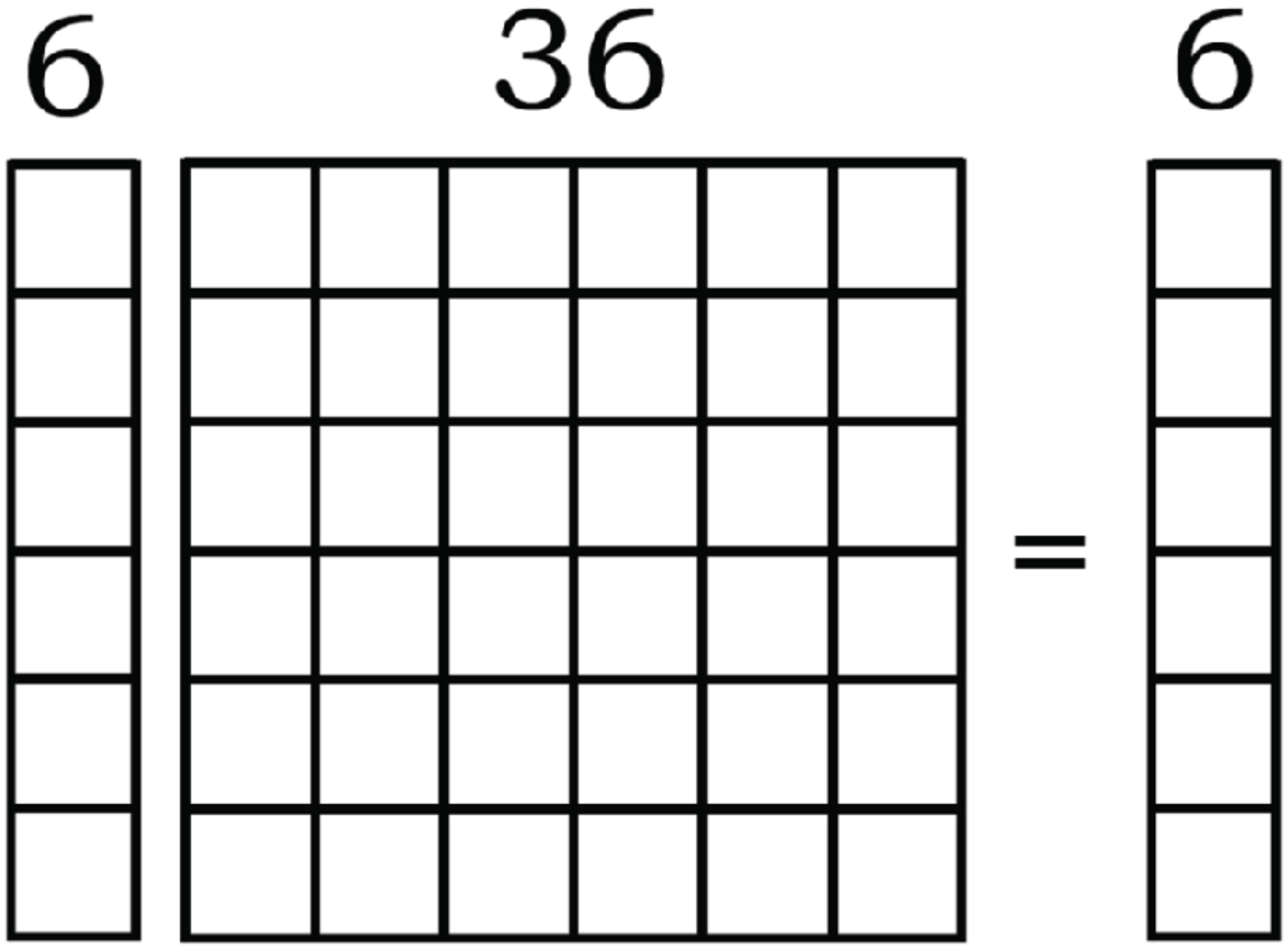
Initially, it appears that we need to know 81 things about the material to specify all elements of the rheology matrix.

2



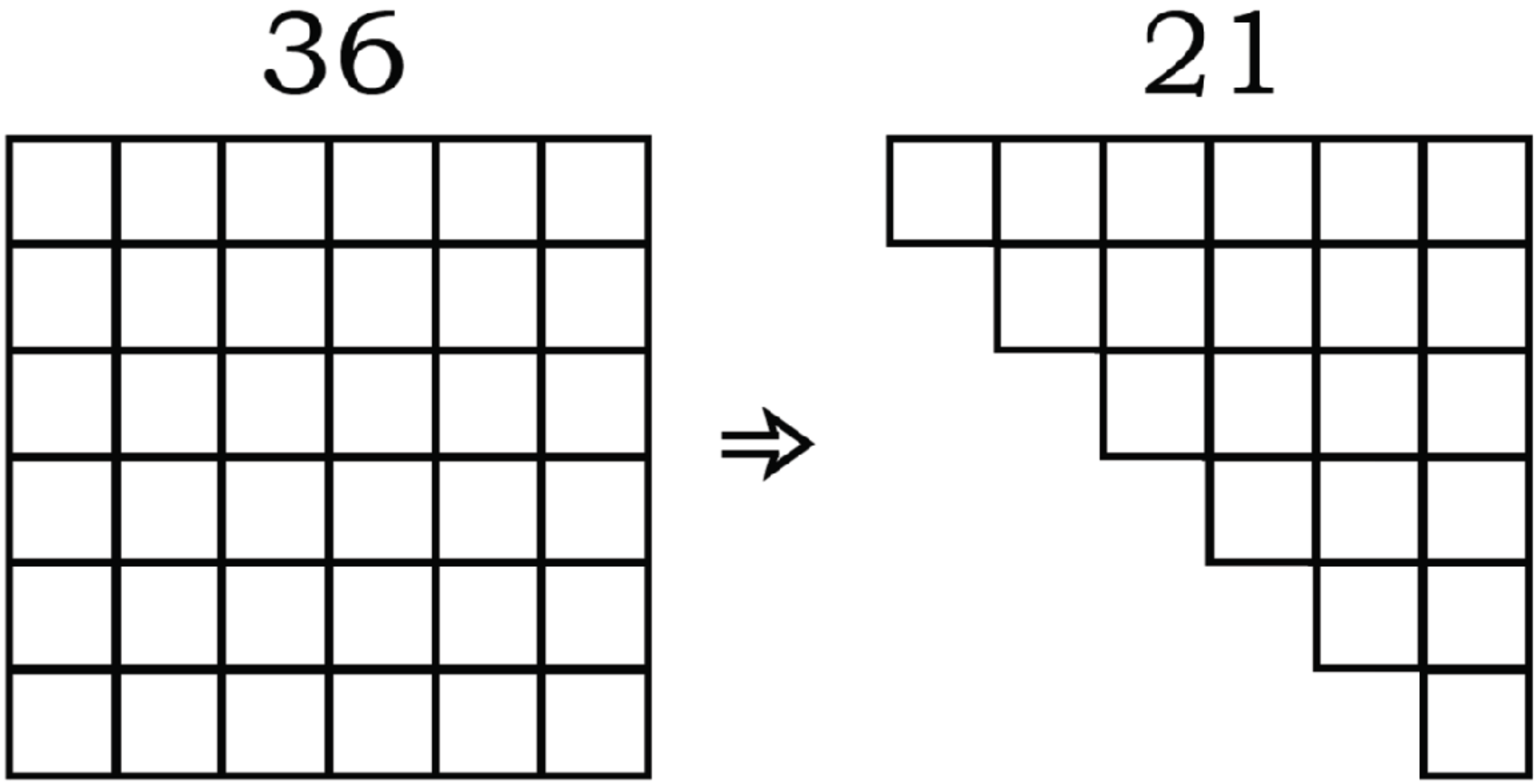
The stress and strain matrices are symmetrical 3x3 matrices, so although they have 9 elements there are only 6 “independent” elements.

3

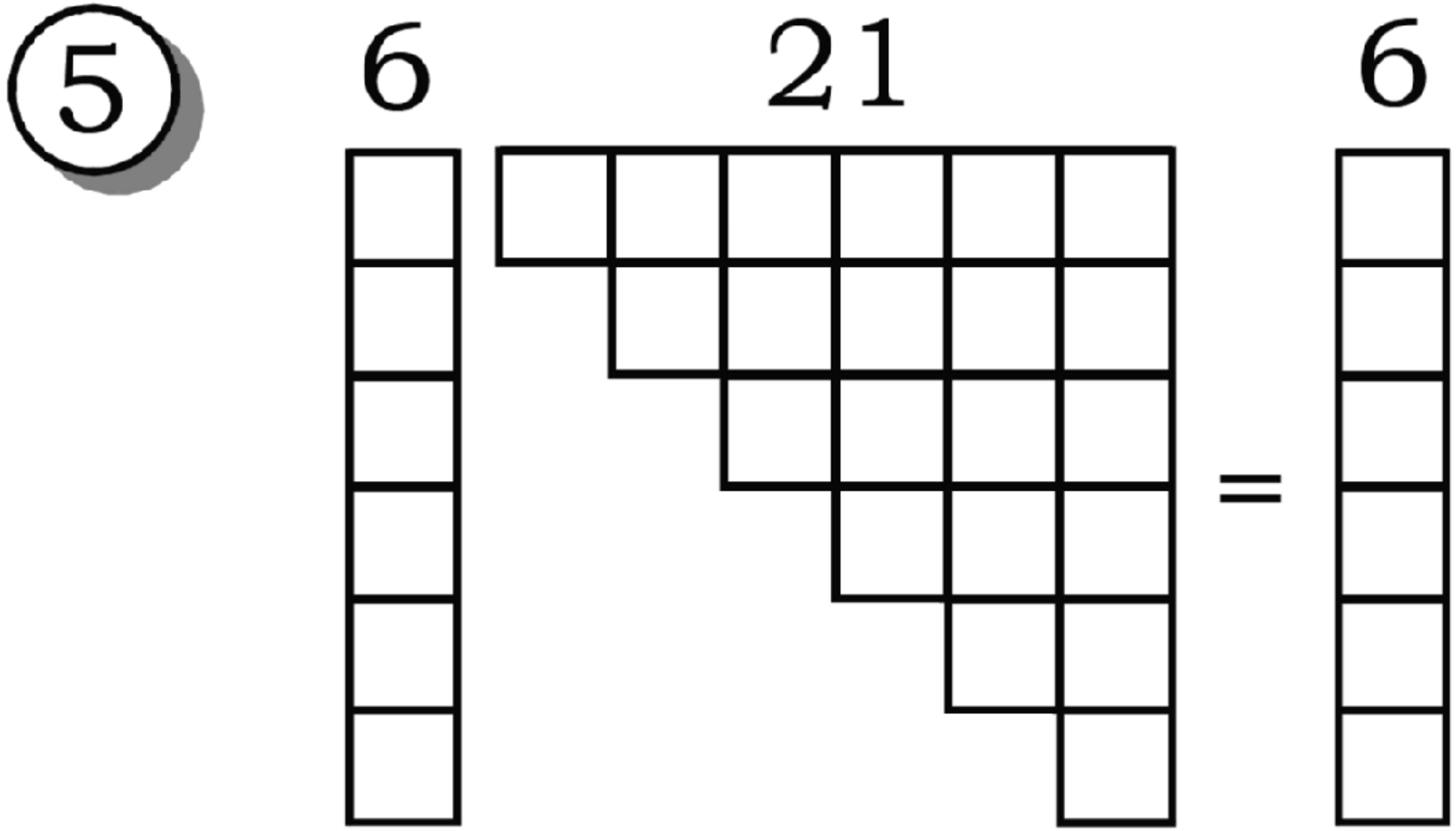


Hence, the rheology matrix only needs to have 36 elements.

4



The rheology matrix is symmetrical, so it has only 21 "independent" elements.



Further simplification is related to the "fabric" of the continuum.

⑥ monoclinic symmetric around z=0 plane

			0	0	
			0	0	
			0	0	
					0
					0

13 nonzero elements

(7) monoclinic and orthotropic

			0	0	0
			0	0	0
			0	0	0
				0	0
					0

9 nonzero elements

⑧ transversely isotropic $\Rightarrow$ 5 elastic constants

$$
\begin{bmatrix}
\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu_z}{E_z} & 0 & 0 & 0 \\
 & \frac{1}{E} & -\frac{\nu_z}{E_z} & 0 & 0 & 0 \\
 & & \frac{1}{E_z} & 0 & 0 & 0 \\
 & & & \frac{1}{2}\mu_z & 0 & 0 \\
 & & & & \frac{1}{2}\mu_z & 0 \\
 & & & & & \frac{1+\nu}{E}
\end{bmatrix}
$$

9. isotropic $\Rightarrow$ 2 elastic constants

$$\begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ & & \frac{1}{E} & 0 & 0 & 0 \\ & & & \frac{2(1+\nu)}{E} & 0 & 0 \\ & & & & \frac{2(1+\nu)}{E} & 0 \\ & & & & & \frac{2(1+\nu)}{E} \end{bmatrix}$$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$$

Poisson's ratio

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$$

Young's modulus

$$\varepsilon_{xy} = \frac{1+\nu}{E}\sigma_{xy}$$

Poisson's ratio

$$\varepsilon_{xy} = \frac{1+\nu}{E}\sigma_{xy}$$

Young's modulus

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{zz} + \sigma_{xx}\right)\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right]$$

$$\varepsilon_{xy} = \frac{1+\nu}{E}\sigma_{xy}$$

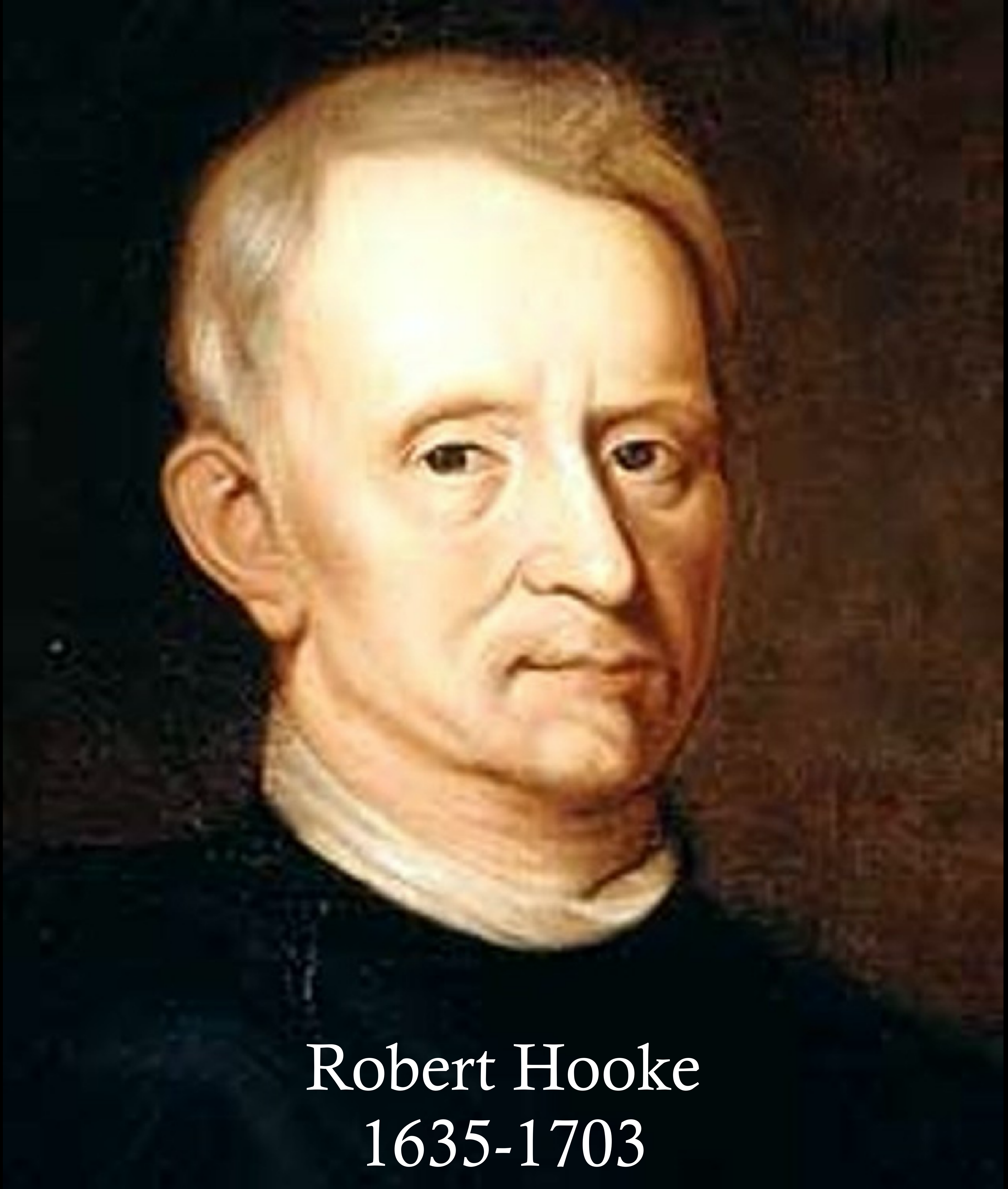
$$\varepsilon_{yz} = \frac{1+\nu}{E}\sigma_{yz}$$

$$\varepsilon_{zx} = \frac{1+\nu}{E}\sigma_{zx}$$

Hooke's Law

stress-strain relationships for a linear-elastic solid

$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$	$\varepsilon_{xy} = \frac{1+\nu}{E}\sigma_{xy}$
$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{zz} + \sigma_{xx}\right)\right]$	$\varepsilon_{yz} = \frac{1+\nu}{E}\sigma_{yz}$
$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right]$	$\varepsilon_{zx} = \frac{1+\nu}{E}\sigma_{zx}$



Robert Hooke
1635-1703

"Only two elastic constants are required for the constituitive equations of the isotropic elastic solid. If the strain components are known, Hooke's law...may be used to calculate the stress components."

David D. Pollard and Stephen J. Martel, 2020, **Structural Geology, A Quantitative Introduction**: Cambridge University Press, p. 134-135.

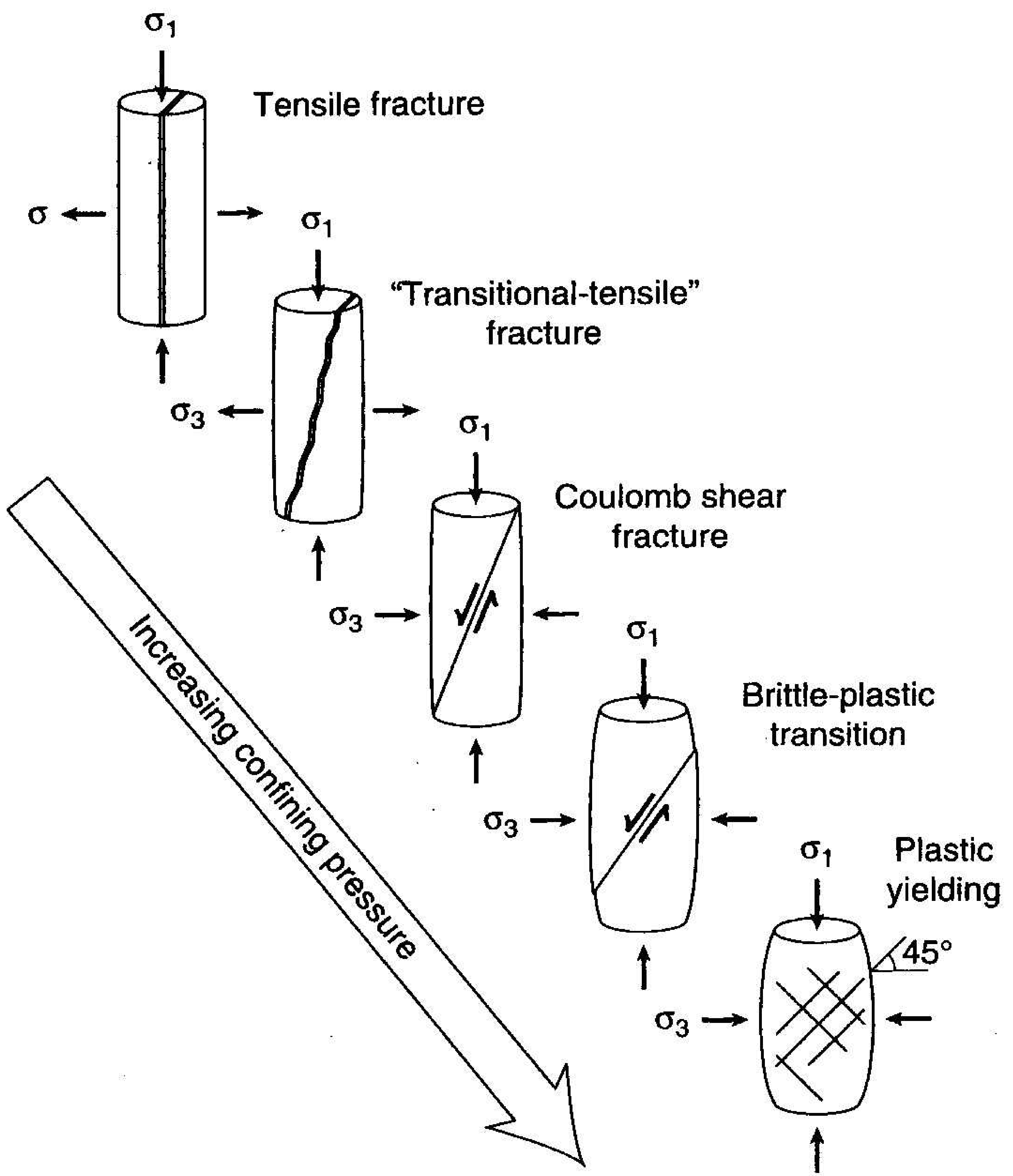
Augustin-Louis Cauchy
1789-1857

David Griggs


Emilio
Segrè
Visual
Archives

Neville Carter


Frank F. Robertson Co.
HOUSTON, TEXAS



σ1
Tensile fracture
σ
σ1
"Transitional-tensile" fracture
σ3
σ1
Coulomb shear fracture
σ3
σ1
Brittle-plastic transition
σ3
σ1
Plastic yielding
45°
σ3
Increasing confining pressure

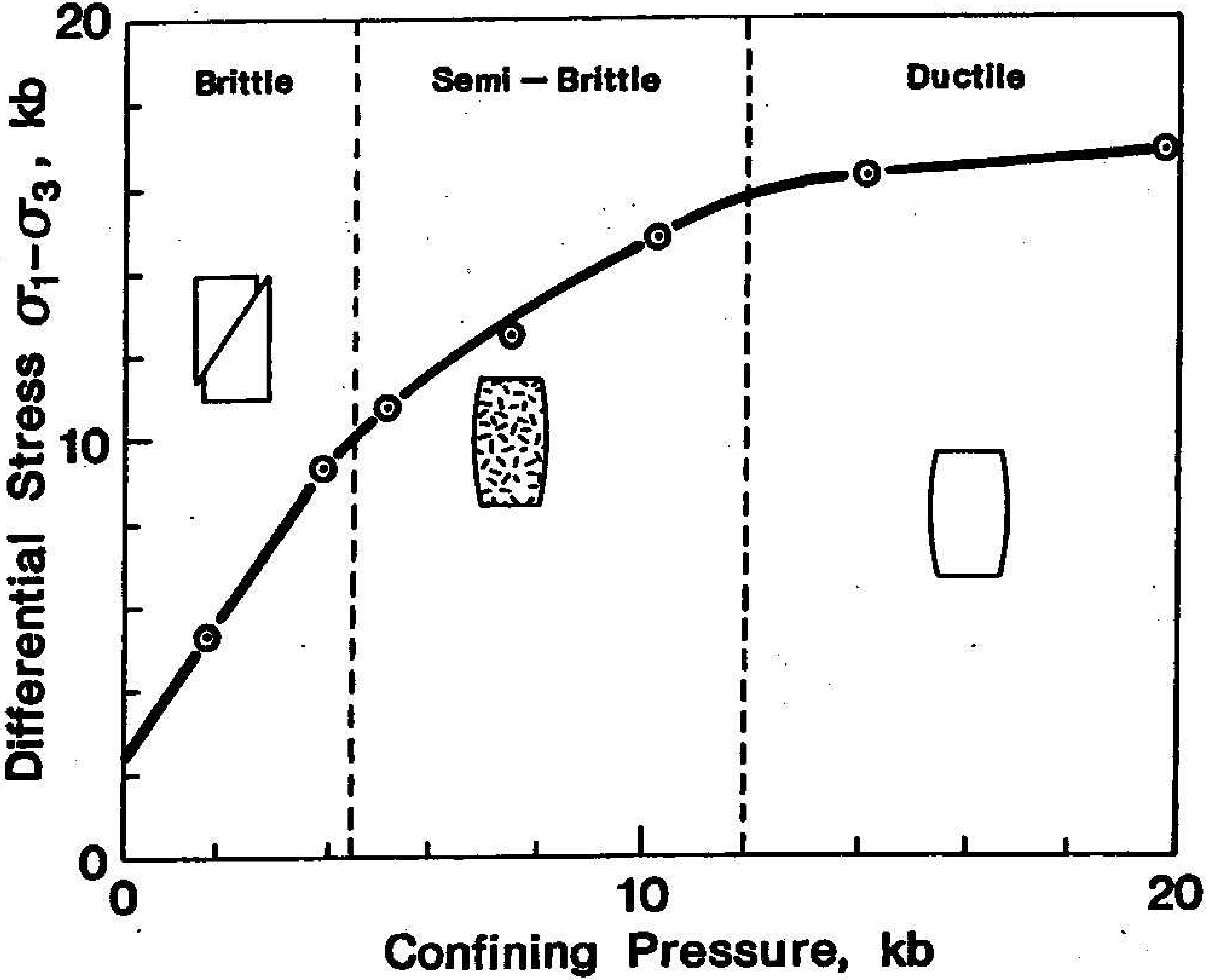
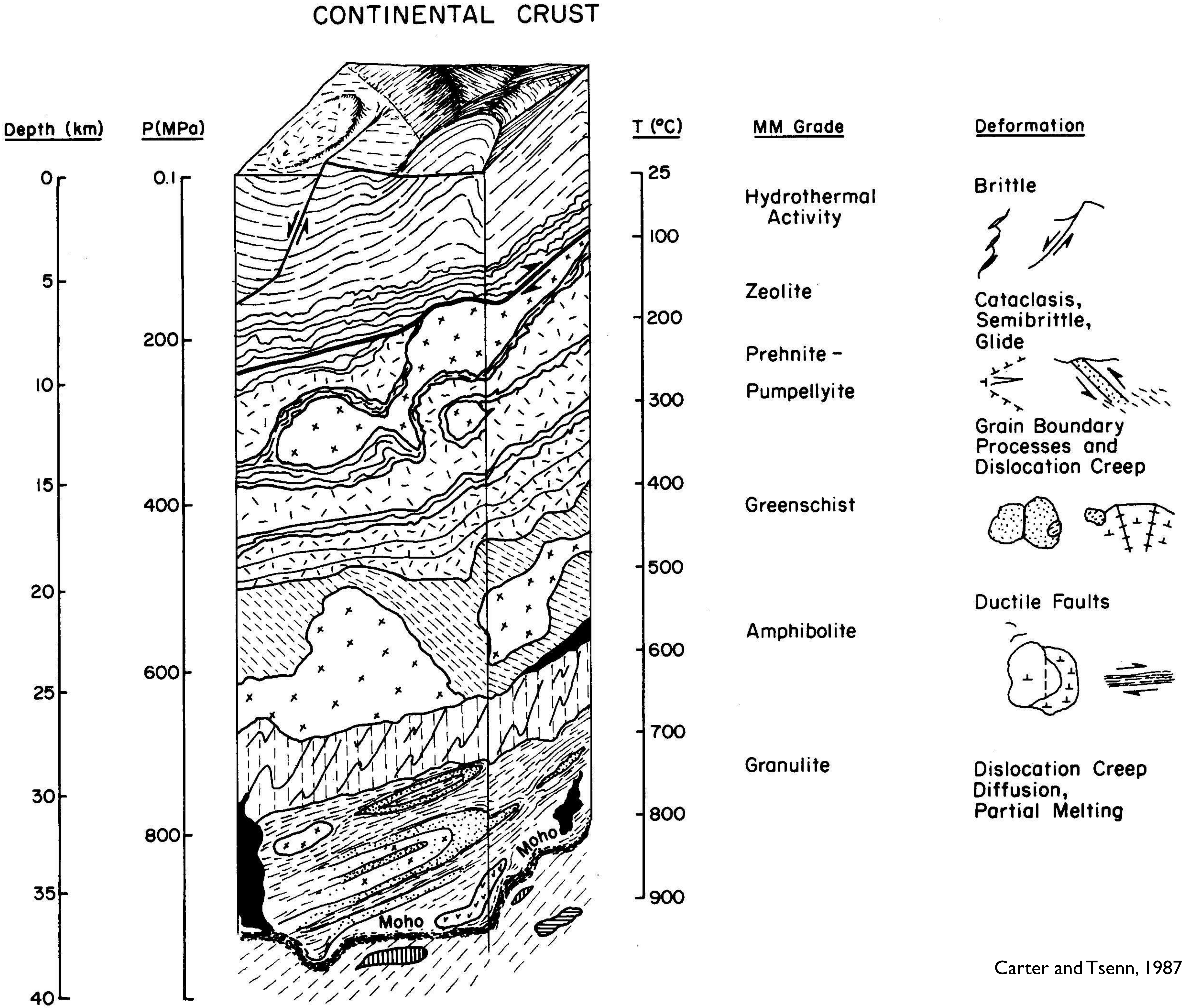
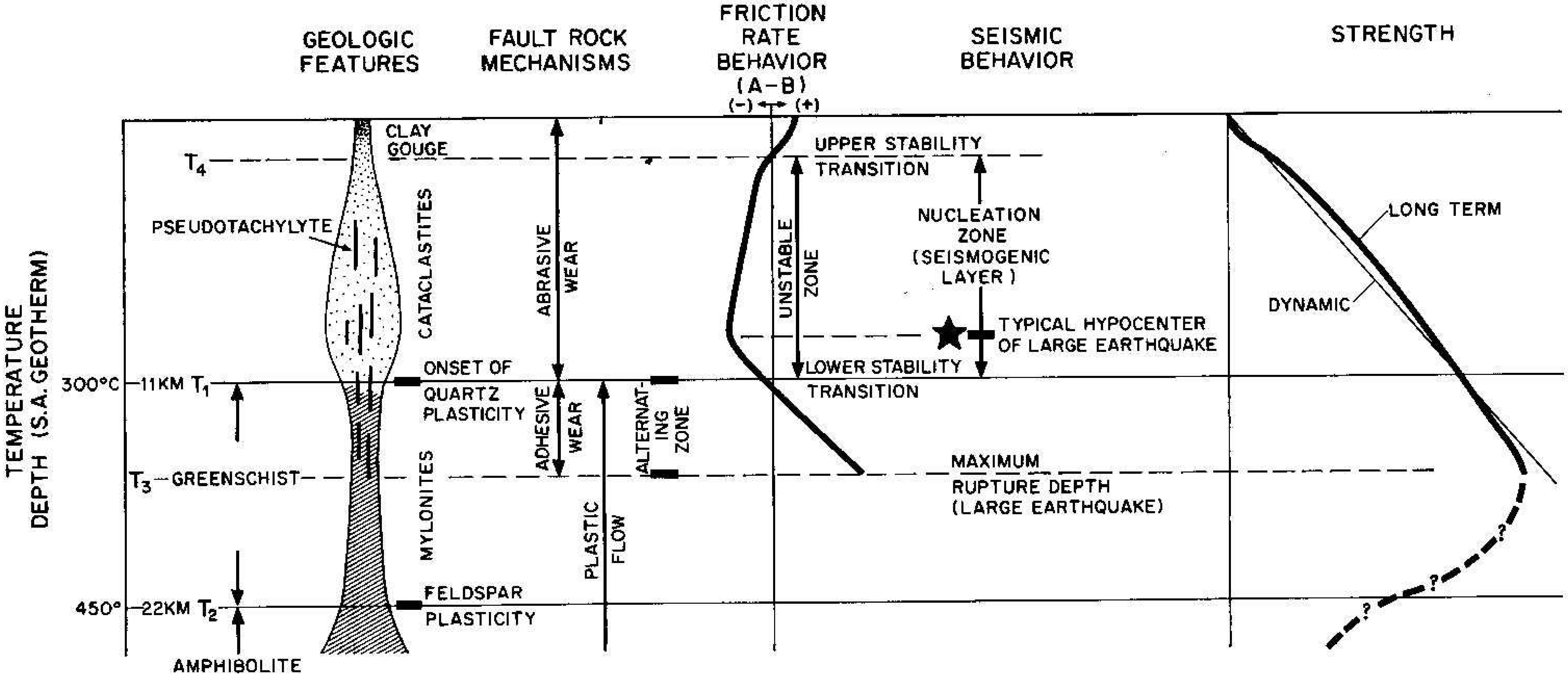


Figure 6-10 Brittle-to-ductile transition of pyroxenite. Effect of confining pressure on the strength of Sleaford Bay clinopyroxenite tested in triaxial compression. *(After Kirby, 1980.)*



Carter and Tsenn, 1987

Seismo-structural section through the San Andreas fault



Scholtz, 1988

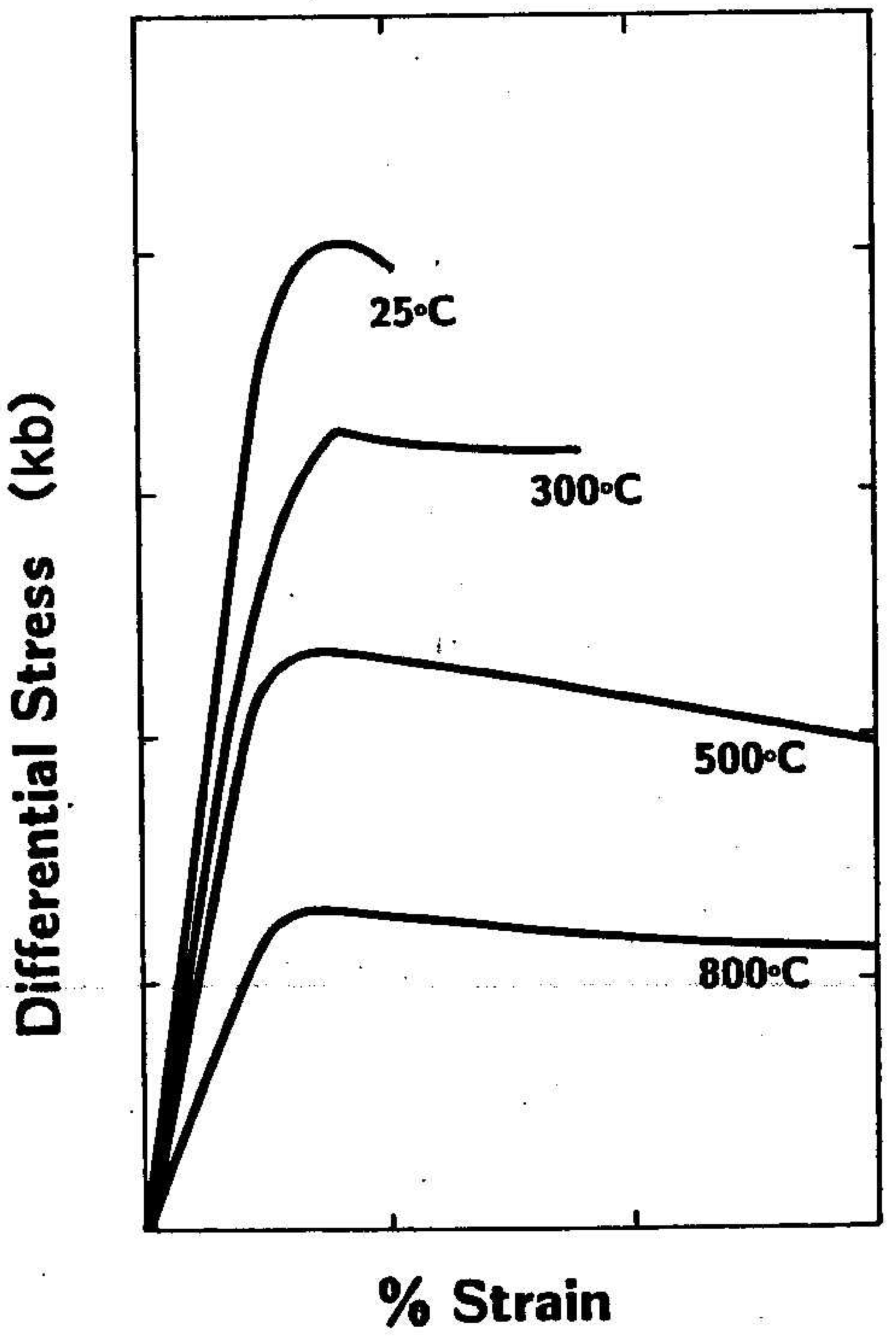


Figure 6-11 Temperature effects on the behavior of granite tested in compression. *(After Heard, 1960.)*

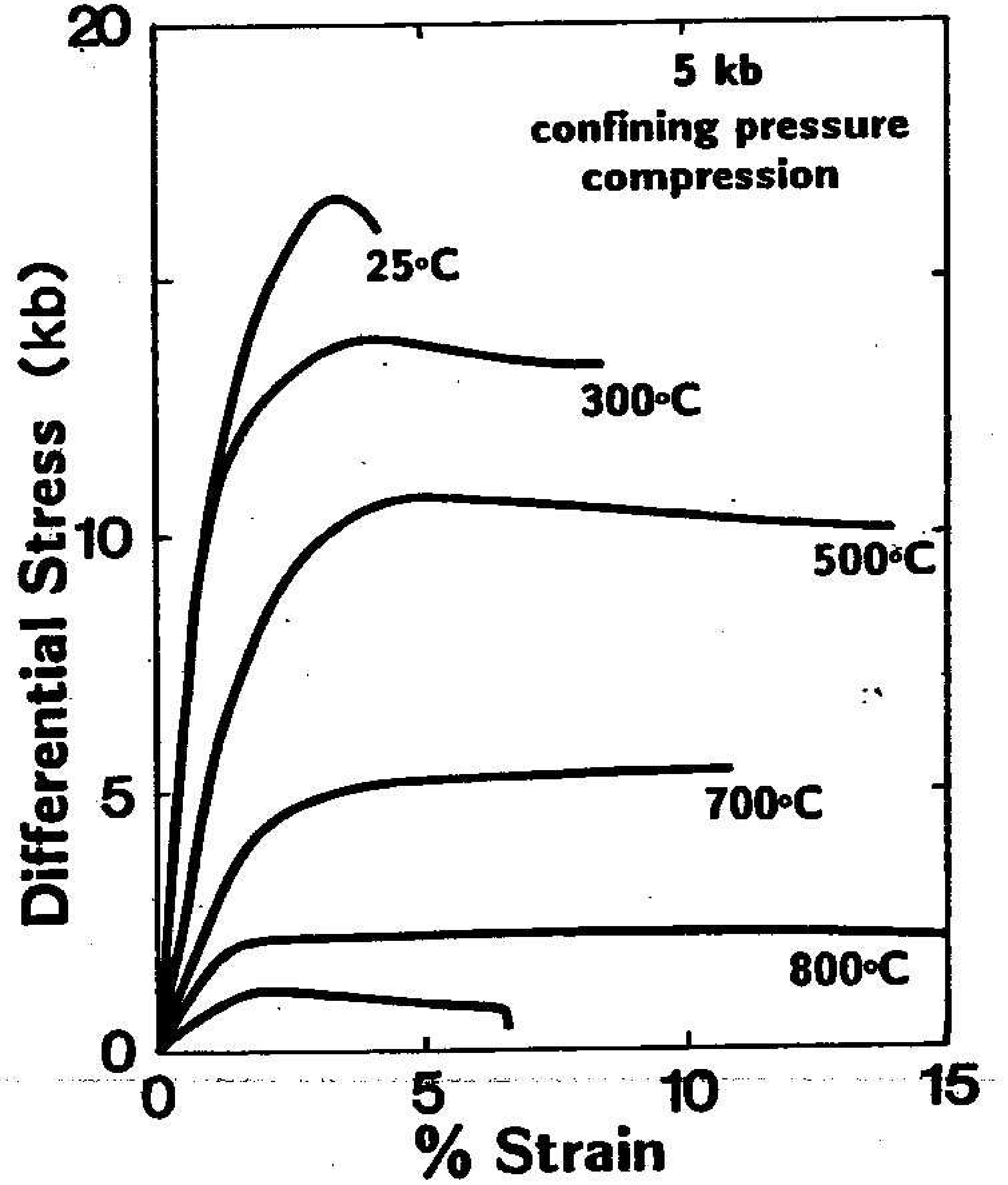
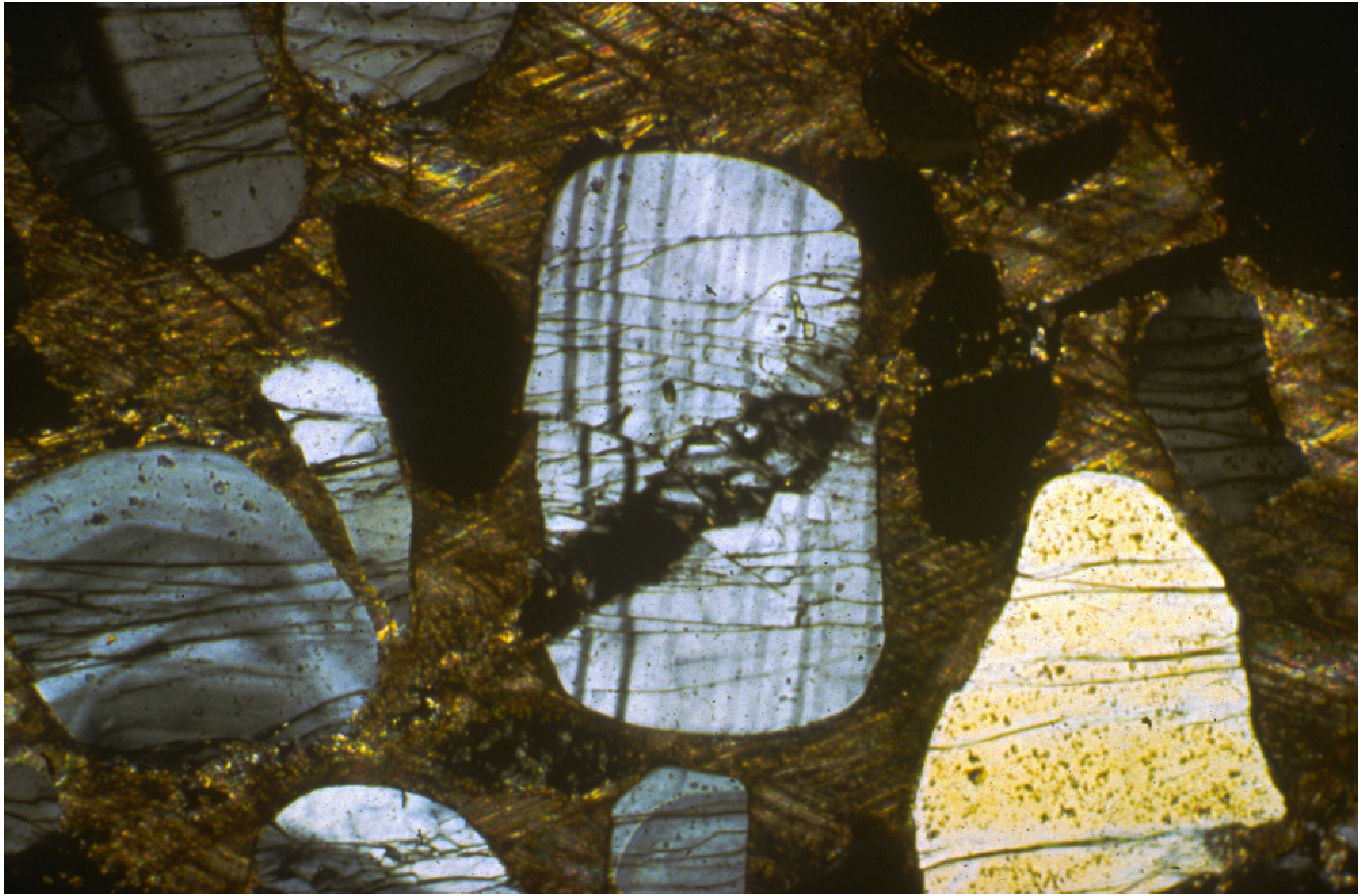


Figure 6-12 Temperature effects on the behavior of basalt tested in compression. *(After Heard, 1960.)*

Brittle deformation in calcite-cemented quartz sandstone.



Texas A&M University Center for Tectonophysics