1-D Extension/Elongation of a Rubber Band

The rubber band has a black mark near one end -- the end that we will attach to a metric ruler using a binder clip. We will make our length measurements relative to that black mark -- that is, the black mark is our ***datum***.

Begin with the rubber band in its shorter state.

When the rubber band is shorter (its initial state), the distance between the datum and the other black mark (l_{bo}) is ___________ cm. (The subscript "bo" stands for "black, original")

... the distance between the datum and the red mark (l_{ro}) is ___________ cm.

Now, carefully stretch the rubber band to its longer state.

When the rubber band is longer (its final state), the distance between the datum and the other black mark (l_{bf}) is ___________ cm. (The subscript "bf" stands for "black, final")

... the distance between the datum and the red mark (l_{rf}) is ___________ cm.

The extension or elongation (e) is

$$e = \frac{l_f - l_o}{l_o}$$

Calculate the extension (e) from the datum to the black mark: e_b = _____________

(2) ... from the datum to the red mark: e_r = ____________

Now we are going to use a very slightly different approach, using vectors. *Note that because this is a 1-dimensional problem, the full specification of the vector is its magnitude (or length) and its positive-or-negative sign.*

The location vectors we need to use are labeled as follows:

- $\mathbf{x}_{bo}$: vector from the datum to the initial position of the black mark (length = l_{bo})
- $\mathbf{x}_{ro}$: vector from the datum to the initial position of the red mark (length = l_{ro})
- $\mathbf{x}_{bf}$: vector from the datum to the final position of the black mark (length = l_{bf})
- $\mathbf{x}_{rf}$: vector from the datum to the final position of the red mark (length = l_{rf})

What is the displacement vector ($\mathbf{u}_b$) between the initial and final positions of the black mark? $\mathbf{u}_b = \mathbf{x}_{bf} - \mathbf{x}_{bo}$ = ________________

What is the displacement vector ($\mathbf{u}_r$) between the initial and final positions of the red mark? $\mathbf{u}_r = \mathbf{x}_{rf} - \mathbf{x}_{ro}$ = ________________

What is the difference in location vectors between the initial position of the red mark and the initial position of the black mark ($\Delta\mathbf{x}_o$)? $\Delta\mathbf{x}_o = \mathbf{x}_{ro} - \mathbf{x}_{bo}$ = _______________

Another way of calculating the extension, using the 1-D vectors described above, is

$$e = \frac{\mathbf{u}_r - \mathbf{u}_b}{\mathbf{x}_{ro} - \mathbf{x}_{bo}} = \frac{\Delta\mathbf{u}}{\Delta\mathbf{x}_o}$$

Calculate the extension using the 1-D vectors: e = _____________

Now we'll do the problem in reverse. *Begin with the rubber band in its **longer** state.*

When the rubber band is longer (its initial state), the distance between the datum and the other black mark (l_{bo}) is ___________ cm.

... the distance between the datum and the red mark (l_{ro}) is ___________ cm.

*Now, carefully relax the rubber band to its **shorter** state.*

When the rubber band is shorter (its final state), the distance between the datum and the other black mark (l_{bf}) is ___________ cm.

... the distance between the datum and the red mark (l_{rf}) is ___________ cm.

The extension or elongation (e) is

$$e = \frac{l_f - l_o}{l_o}$$

Calculate the extension (e) from the datum to the black mark: e_b = _____________

(2) ... from the datum to the red mark: e_r = ____________

As before, the location vectors we need to use are labeled as follows:

$\mathbf{x}_{bo}$: vector from the datum to the initial position of the black mark (length = l_{bo})

$\mathbf{x}_{ro}$: vector from the datum to the initial position of the red mark (length = l_{ro})

$\mathbf{x}_{bf}$: vector from the datum to the final position of the black mark (length = l_{bf})

$\mathbf{x}_{rf}$: vector from the datum to the final position of the red mark (length = l_{rf})

What is the displacement vector ($\mathbf{u}_b$) between the initial and final positions of the black mark? $\mathbf{u}_b = \mathbf{x}_{bf} - \mathbf{x}_{bo}$ = ________________

What is the displacement vector ($\mathbf{u}_r$) between the initial and final positions of the red mark? $\mathbf{u}_r = \mathbf{x}_{rf} - \mathbf{x}_{ro}$ = ________________

What is the difference in location vectors between the initial position of the red mark and the initial position of the black mark ($\Delta\mathbf{x}_o$)? $\Delta\mathbf{x}_o = \mathbf{x}_{ro} - \mathbf{x}_{bo}$ = _______________

Another way of calculating the extension, using the 1-D vectors described above, is

$$e = \frac{\mathbf{u}_r - \mathbf{u}_b}{\mathbf{x}_{ro} - \mathbf{x}_{bo}} = \frac{\Delta\mathbf{u}}{\Delta\mathbf{x}_o}$$

Calculate the extension using the 1-D vectors: e = _____________

When the length of the rubber band is *increased*, the sign of the extension is _______________, but when the rubber band is shortened, the sign of the extension is ________________.

The extension is the same as the 1-D displacement gradient.