Chapter 8. Instantaneous plate kinematics in an imaginary 3-plate world

8.1 Introduction

In the genesis story of plate kinematics, the continental reconstruction paper of Bullard, Everett and Smith (1965) introduced the idea of an Euler pole, named for Leonhard Euler who originated what has come to be called the fixed-point theorem for motion of a spherical shell over a sphere (Euler, 1776). We worked with Euler poles or poles of rotation in the previous 4 chapters. The paper of McKenzie and Parker (1967) introduced the idea of the angular velocity vector, also known as a rotation vector or an axial vector. McKenzie and Morgan (1969) further developed the idea, and McKenzie and Slater (1971) used the basic ideas of spherical kinematics to investigate the opening of the Indian Ocean and the breakup of Gondwananland. By 1973 when LePichon, Francheteau and Bonnin published their textbook ***Plate Tectonics*** and Allan Cox published his compilation of key papers, the first-generation understanding of plate kinematics had been established. Just before his death, Cox published a textbook in which he attempted an elementary explanation of that first-generation kinematic model (Cox and Hart, 1986).

In this chapter, we will work to understand the basics of angular velocity vectors in the context of 2+ and 3-plate systems. In a 2+plate system, we observe the motion of two plates relative to each other and relative to a frame of reference external to the two plates. In a 3-plate system, we observe the motion of three plates relative to each other. We will leave this chapter armed with what we need to know to understand the basic kinematics of a tectonic system that includes any number of more-or-less rigid plates.

8.2 A user-defined function

We will use the following user-defined function developed in a previous chapter.

```
In[1]:= convert2Cart[lat_, long_] := {Cos[lat Degree] Cos[long Degree],
          Cos[lat Degree] Sin[long Degree], Sin[lat Degree]};
```

8.3 Instantaneous kinematics in a 3-plate system

Quick reminder about vector arithmetic

First, let's get our arrow nomenclature straight, given that we use arrows to visualize vectors. The pointy end of an arrow is called the tip or head, and the other end with the feathers is called the nock.

Consider two vectors, **a** and **b**, as shown in Figure 8-1. Positive vector **a** points in one direction, and negative vector **a** has the same magnitude (length) but points in the opposite direction, 180° away from **a**. Consequently, the sum **a** + (−**a**) = 0. The sum of vectors **a** and **b** is vector **c**, and is visualized by touching the tip of vector **a** to the nock of vector **b**. Vector **c** extends from the nock of vector **a** to the tip of vector **b**. The result of subtracting vector **b** from vector **a** is equivalent to adding vector **−b** to **a**, yielding vector **d**. Following the same logic, vector **e** is the difference between **b** and **a**, and vector **f** is the difference between **−b** and **a**.

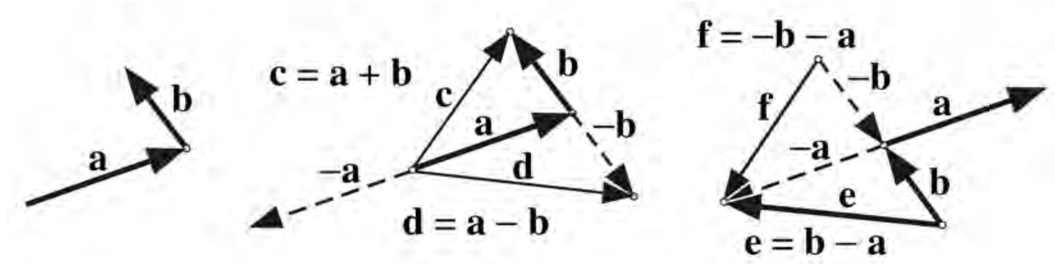

Figure 8-1. Visualization of some simple arithmetic operations using vectors.

Let's code-up a few vectors and make sure we understand what is going on. First, we will initialize or define an arbitrary vector called **a**.

```
In[2]:= a = {1, 2, 3};
```

The components of the negative of vector **a** (that is, of **−a**) should be the negative values of the components of vector **a**.

```
In[3]:= -a

Out[3]= {-1, -2, -3}
```

The same should be true of an arbitrary vector **b** and its negative, **-b**.

```
In[4]:= b = {2, 3, 4};

In[5]:= -b

Out[5]= {-2, -3, -4}
```

When two vectors are added together, the sum is formed by adding their respective components together.

That is, **a** + **b** = {(**a**[[1]]+**b**[[1]]), (**a**[[2]]+**b**[[2]]), (**a**[[3]]+**b**[[3]])}.

```
In[6]:= a + b

Out[6]= {3, 5, 7}
```

And when one vector is subtracted from another, the difference is formed by subtracting the corresponding components.

That is, **a** −**b** = {(**a**[[1]]-**b**[[1]]), (**a**[[2]]-**b**[[2]]), (**a**[[3]]-**b**[[3]])}.

```
In[7]:= a - b

Out[7]= {-1, -1, -1}
```

Linear velocity vector circuits

Imagine that you are no longer your usual corporeal self, but rather a point in space. Your name is A. Nearby, you notice two other distinct points B and C, whose positions are such that the three of you are not located on the same line (Figure 8-2). You notice that these two other points are moving in different directions relative to you. At any instant in time, you can specify a vector that describes the instantaneous motion of point B relative to you: ${}_AV_B$.

(I am using the same nomenclature that McKenzie and Parker used in 1967, which has the form ${}_{\text{frame of reference}}V_{\text{moving object}}$. While this should be the convention used by all in plate kinematics, sadly it seems that different workers have used different and sometimes opposite nomenclature. Beware!)

Meanwhile, your friend on point B also notices the motion of points A and C relative to her frame of reference. Relative to point B, point A has an instantaneous velocity of ${}_BV_A$, and point C's instantaneous velocity is ${}_BV_C$. And on point C, the instantaneous velocity of point A is ${}_CV_A$, and point B is moving at a rate of ${}_CV_B$.

It is probably obvious that the vector for the instantaneous velocity of point B as observed from point A is equal in magnitude but opposite in direction from the vector for the instantaneous velocity of point A as observed from point B. That is,

$$|{}_AV_B| = |{}_BV_A| \text{ and } {}_AV_B = -{}_BV_A$$

And it follows that

$$|{}_BV_C| = |{}_CV_B| \text{ and } {}_BV_C = -{}_CV_B$$

$$|{}_AV_C| = |{}_CV_A| \text{ and } {}_AV_C = -{}_CV_A$$

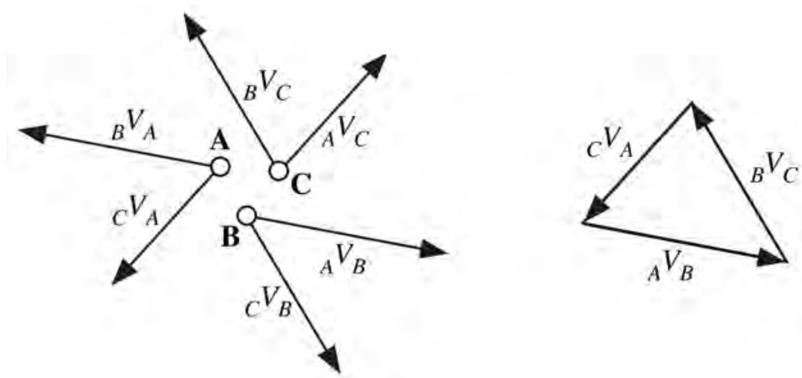

Figure 8-2. One of an infinite number of vector scenarios involving three points (A, B and C) that are in motion relative to each other at an instant in time. The instantaneous relative motion vectors defined at each point are shown at left, and a corresponding closed vector circuit is shown at right.

If you pooled your observations with your friends on points B and C for a given instant, you would discover that

$${}_AV_B + {}_BV_C + {}_CV_A = 0$$

and all three instantaneous velocity vectors would lie on plane ABC. Given this scenario of relative velocities defined at three points at an instant in time, the only reason the velocities would not sum to zero is if one or more of the velocities is not accurate.

A set of vectors that sums to zero is called a closed vector circuit, and the vectors in this particular circuit can be envisioned as forming the sides of a triangle. A triangle formed by three line segments (as contrasted with a spherical triangle) is a plane figure, meaning that all of its elements are contained within a specific plane. If the three vectors did not achieve closure, they would not necessarily be coplanar. If the three vectors are not coplanar, they cannot achieve closure.

Take three sharpened pencils as analogs for our vectors. Place the pencils point-to-eraser so that they form a closed triangle on the table top (Figure 8-3A). The vertices of the pencil triangle are the three points that are moving with respect to one another. This configuration of pencils is like our equation above, in which the three vectors sum to zero. They achieve closure.

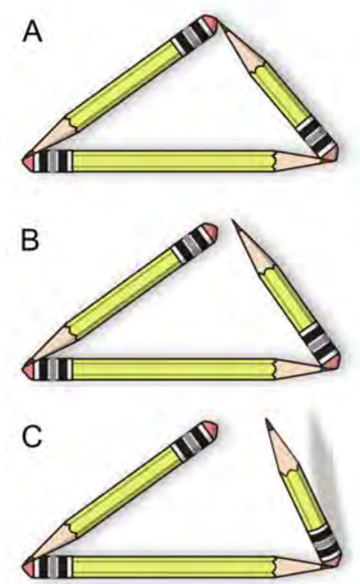

Figure 8-3. Pencils serving as analogs for vectors. (A) Closed vector circuit. (B) Coplanar vectors that do not form a closed vector circuit. (C) Non-coplanar vectors cannot form a vector circuit.

Now, move the pencils while still on the table so that the three pencils are still touching point-to-eraser in two places, but create a gap where the third vertex had been (Figure 8-3B). Here, the three pencils are still coplanar, but there is not closure. Having a gap like that would not make sense, because the eraser on one pencil and the point on the other pencil across the gap correspond to the same point, not two points separated by a gap.

Now rotate one of the pencils adjacent to the opening and lift one end so that it is not touching the table, but its other end is still touching the next pencil (Figure 8-3C). This models three non-coplanar vectors, which cannot achieve closure. Again, what should be one point is two points separated by a gap. In order for three vectors to be part of a closed vector circuit, they must be coplanar.

One of the important realizations related to a vector circuit is that if you only know two of the vectors, the third vector can be computed. So imagine that you know the velocity of point B from your vantage point, point B reports the velocity of point C from her vantage point, but there is a fog that obscures point C from you. The unknown velocity is given by

$$ {}_AV_B + {}_BV_C = {}_AV_C $$

This makes sense, because if we add ${}_CV_A$ to both sides of the previous equation, we get

$$ ({}_AV_B + {}_BV_C) + {}_CV_A = ({}_AV_C) + {}_CV_A $$

and

$$ {}_AV_C + {}_CV_A = 0 $$

because

$$ {}_AV_C = -{}_CV_A $$

which returns us to

$$ {}_{A}V_{B} + {}_{B}V_{C} + {}_{C}V_{A} = 0 $$

Angular velocity vector circuits

In the last several chapters, we have worked with the idea of rotating a point or a set of points around an axis. In plate kinematics using a spherical model Earth, the axis of rotation passes through the center of the sphere and intersects the outer surface of the sphere at points we call poles. The positive pole corresponds to a right-handed rotation as observed looking down on the surface of the sphere. We will define an angular velocity vector as a vector whose magnitude is the angular speed, whose direction is colinear with the rotation axis, and whose sign is determined by the right-hand convention. McKenzie and Parker (1967) explained it differently, but equivalently, as follows: "The sign convention takes a rotation which is clockwise when looked at from the centre of the sphere to be a positive vector which is pointing outward along the rotation axis."

The instantaneous kinematics of any three-plate system is governed by three angular-velocity vectors. Like their linear-motion counterparts, the three angular velocity vectors are coplanar, and have a simple relationship with each other given by

$$ {}_{A}\Omega_{B} + {}_{B}\Omega_{C} + {}_{C}\Omega_{A} = 0 $$

where we will generally use the capital omega (Ω) to indicate the full angular-velocity vector and the small-case omega (ω) to indicate angular speed -- the magnitude of the angular velocity vector. Just as with the linear velocity vectors, the instantaneous angular velocity vectors form a vector circuit. And as with the linear velocity vectors, if you know two of the angular velocity vectors, you can compute the third.

Extending a location vector along a given angular velocity vector, we identify a point on Earth's surface called the pole. And so, if we find a location vector coincident with ${}_{A}\Omega_{B}$, we might refer to that pole symbolically as ${}_{\mathrm{A}}P_{\mathrm{B}}$, meaning the pole around which plate B rotates in a positive (counter-clockwise) direction as observed from plate A

The origin of all three angular velocity vectors is the same: the center of our model Earth. Hence, the intersection of the plane that contains all three angular velocity vectors and the surface of our spherical model Earth is a great circle. This leads us to one of the more important observations in plate kinematics: the three poles of instantaneous relative motion for any 3-plate system are located on the same great circle. But where, relative to each other?

Let's work on this problem of where the poles are for a given three-plate system by considering relative motions at the simplest location to do so: at one of the two points that are 90° away from the great circle. As we saw in the last chapter, it is easy to compute the tangential velocity and direction of motion at a point that is 90° away from a pole. We can make the process even easier if we specify a model Earth that has a radius of 1. (One *what*, you ask? The unit we are using is an Earth radius (R_{E}), which is equivalent to ~6371 km).

Imagine three plates, labeled A, B and C, with the following instantaneous kinematic data.

Frame of Reference	**Moving Plate**	**Angular Speed** $\left(°/\mathrm{Myr}\right)$	**Pole Location** $(\mathrm{Lat},\ \mathrm{Long})$
A	B	0.852	−48.77, 106.09
B	C	2.208	38.72, −107.39
C	A	1.489	−29.80, 58.72

On a model Earth with a radius of 1, the tangential velocity of plate B relative to plate A at a point that is 90° from the pole (${}_{\mathrm{A}}V_{\mathrm{B}}$) is equal to 0.852 R_{E}.

$$ {}_{\mathrm{A}}V_{\mathrm{B}} = 0.852\ R_{\mathrm{E}} $$

Similarly,

$$ {}_{\mathrm{B}}V_{\mathrm{C}} = 2.208\ R_{\mathrm{E}} $$
$$ {}_{\mathrm{C}}V_{\mathrm{D}} = 1.489\ R_{\mathrm{E}} $$

All of the velocities are defined at the same point, and all are tangential to our spherical model Earth. Consequently, all of these velocity vectors are coplanar. We know that the corresponding vector circuit must be closed

$$ {}_{A}V_{B} + {}_{B}V_{C} + {}_{C}V_{A} = 0 $$

so we can visualize the problem by creating a triangle that has sides with lengths of 0.852, 2.208 and 1.489, corresponding to ${}_{A}V_{B}$, ${}_{B}V_{C}$ and ${}_{C}V_{A}$, respectively (Figure 8-4). The angle between the sides with lengths of 0.852 and 2.208 is related to the angle between ${}_{A}V_{B}$ and ${}_{B}V_{C}$, and hence related to the angle between ${}_{A}\Omega_{B}$ and ${}_{B}\Omega_{C}$. The angle between ${}_{A}\Omega_{B}$ and ${}_{B}\Omega_{C}$ is the same as the angle between the corresponding two rotational poles, ${}_{\mathrm{A}}P_{\mathrm{B}}$ and ${}_{\mathrm{B}}P_{\mathrm{C}}$, along the great circle. So it seems that the instantaneous angular speeds (the unsigned magnitudes of the instantaneous angular velocity vectors) determine the relative positions of the poles of instantaneous rotation.

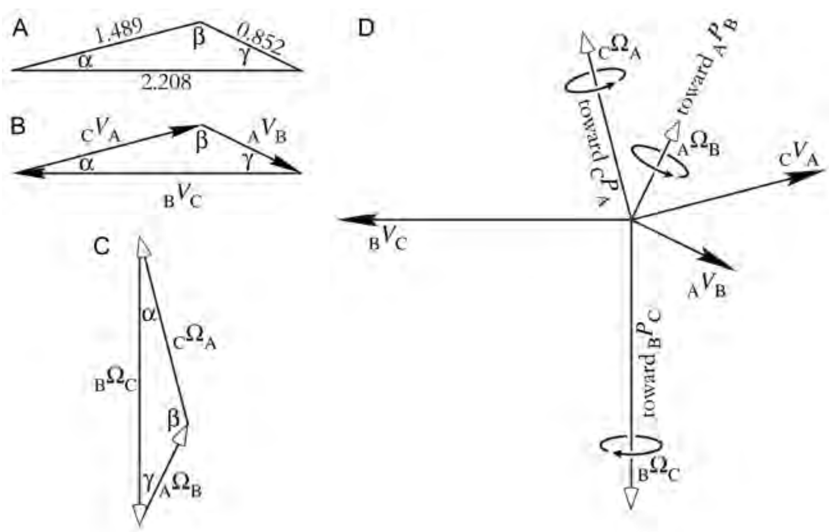

Figure 8-4. Instantaneous vector relationships for a given 3-plate system. (A) Triangle with sides scaled to the angular speeds. (B) Tangential velocity vectors forming a closed vector circuit. (C) Positive angular velocity vectors forming a closed vector circuit. (D) Spatial relationship of positive angular velocity vectors and tangential velocity vectors at a point located 90° from the poles of rotational motion. Each angular velocity vector is perpendicular to the corresponding tangential velocity vector.

So what are these interior angles, given sides with lengths of 0.852, 2.208 and 1.489? We can use a form of the cosine law from trigonometry to find the interior angles. For a triangle with sides whose lengths are a, b and c and whose interior angles are α (angle opposite side a and between sides b and c), β (angle opposited side b...) and γ (angle opposite side c...), the corresponding equations are

$$ \alpha = \arccos\left(\frac{b^2 + c^2 - a^2}{2\,b\,c}\right) $$

$$\beta = \arccos\left(\frac{a^2 + c^2 - b^2}{2\,a\,c}\right)$$

$$\gamma = \arccos\left(\frac{a^2 + b^2 - c^2}{2\,a\,b}\right)$$

Let's code it up and get some results, expressed in degrees.

```
In[8]:= a = 0.852; b = 2.208; c = 1.489;

In[9]:= α = ArcCos[(b^2 + c^2 - a^2) / (2 * b * c)] / Degree

Out[9]= 14.4826

In[10]:= β = ArcCos[(a^2 + c^2 - b^2) / (2 * a * c)] / Degree

Out[10]= 139.601

In[11]:= γ = ArcCos[(a^2 + b^2 - c^2) / (2 * a * b)] / Degree

Out[11]= 25.9167
```

The data table above contains data from the RM2 model of Minster and Jordan (1978), which was the most widely used kinematic dataset until the publication of the Nuvel-1 model by Demets and others in 1990. Let's see how our results agree with the angular distance between the RM2 poles.

```
In[12]:= aPoleB = convert2Cart[-48.77, 106.09];

In[13]:= bPoleC = convert2Cart[38.72, -107.39];

In[14]:= cPoleA = convert2Cart[-29.80, 58.72];
```

The angle between the ${}_{B}P_{C}$ pole and the ${}_{C}P_{A}$ pole is

```
In[15]:= bPc2cPa = VectorAngle[bPoleC, cPoleA] / Degree

Out[15]= 165.491
```

Between the ${}_{A}P_{B}$ pole and the ${}_{C}P_{A}$ pole, the angle is

```
In[16]:= aPb2cPa = VectorAngle[aPoleB, cPoleA] / Degree

Out[16]= 40.4383
```

And the angle between pole ${}_{B}P_{C}$ and pole ${}_{A}P_{B}$ is

```
In[17]:= bPc2aPb = VectorAngle[bPoleC, aPoleB] / Degree

Out[17]= 154.071
```

It is probably worthwhile to notice the following:

```
In[18]:= α + bPc2cPa

Out[18]= 179.974
```



```
In[19]:= β + aPb2cPa
Out[19]= 180.039
In[20]:= γ + bPc2aPb
Out[20]= 179.987
```

We previously asserted that the angles inside the vector triangle are related to the angles between the poles, and also to the angles between the angular velocity vectors. In the case we have investigated, the interior angles of the plane triangle are the supplementary angles of the angles between the poles. (Two angles are supplementary if they sum to 180°.) That is, the angle between ${}_{B}P_{C}$ and ${}_{C}P_{A}$ is (180 - α). Let's check.

```
In[21]:= bPc2cPa
Out[21]= 165.491
In[22]:= 180 - α
Out[22]= 165.517
In[23]:= mismatch1 = Abs[bPc2cPa - %]
Out[23]= 0.026464
```

What about the rest?

```
In[24]:= aPb2cPa
Out[24]= 40.4383
In[25]:= 180 - β
Out[25]= 40.3993
In[26]:= mismatch2 = Abs[aPb2cPa - %]
Out[26]= 0.0390307
In[27]:= bPc2aPb
Out[27]= 154.071
In[28]:= 180 - γ
Out[28]= 154.083
In[29]:= mismatch3 = Abs[bPc2aPb - %]
Out[29]= 0.0125667
```

Based on the mismatches computed above, the published answers and our computed answers agree within a tenth of a degree. We conclude that the relative positions of the three positive poles of instantaneous rotation for any three-plate system are controlled by the corresponding angular speeds. As each negative pole of rotation is the antipode of a positive pole (that is, it is 180° away from the positive pole), the relative position of all of the poles of instantaneous rotation for any three-plate system are determined by the respective angular speeds.

Boiling out the fat to solve another demonstration problem

Given the following data from the RM2 model of Minster and Jordan (1978), derive the angles between the poles from the angular velocity data, and check the results against the published pole locations.

Frame of Reference	Moving Plate	Angular Speed (°/Myr)	Pole Location (Lat, Long)
A	B	0.356	66.56, −37.29
B	C	0.149	−9.46, 138.30
C	A	0.302	−87.69, −104.80

```
In[30]:= aωb = 0.356;

In[31]:= bωc = 0.149;

In[32]:= cωa = 0.302;

In[33]:= aPb = convert2Cart[66.56, -37.29];

In[34]:= bPc = convert2Cart[-9.46, 138.30];

In[35]:= cPa = convert2Cart[-87.69, -104.80];

In[36]:= α = ArcCos[(bωc^2 + cωa^2 - aωb^2) / (2 * bωc * cωa)] / Degree;

In[37]:= β = ArcCos[(aωb^2 + cωa^2 - bωc^2) / (2 * aωb * cωa)] / Degree;

In[38]:= γ = ArcCos[(aωb^2 + bωc^2 - cωa^2) / (2 * aωb * bωc)] / Degree;

In[39]:= bPc2cPa = 180 - α;

In[40]:= VectorAngle[bPc, cPa] / Degree;

In[41]:= mismatch1 = Abs[bPc2cPa - %]

Out[41]= 0.109555

In[42]:= aPb2cPa = 180 - β;

In[43]:= VectorAngle[aPb, cPa] / Degree;

In[44]:= mismatch2 = Abs[aPb2cPa - %]

Out[44]= 0.039637

In[45]:= aPb2bPc = 180 - γ;

In[46]:= VectorAngle[aPb, bPc] / Degree;

In[47]:= mismatch3 = Abs[aPb2bPc - %]

Out[47]= 0.149193
```

Judging from the mismatches computed above the computed angles between the poles is within ~0.1° of the published angles. The computed angle (in degrees) between the ${}_{\mathrm{A}}P_{\mathrm{B}}$ pole and the ${}_{\mathrm{B}}P_{\mathrm{C}}$ pole is

In[48]:= **aPb2bPc**

Out[48]= 122.97

Between the ${}_{\mathrm{B}}P_{\mathrm{C}}$ pole and the ${}_{\mathrm{C}}P_{\mathrm{A}}$ pole, the angle is

In[49]:= **bPc2cPa**

Out[49]= 81.4815

And the angle between pole ${}_{\mathrm{C}}P_{\mathrm{A}}$ and pole ${}_{\mathrm{A}}P_{\mathrm{B}}$ is

In[50]:= **aPb2cPa**

Out[50]= 155.549

Why don't the data from Minster and Jordan (1978) agree with our analysis perfectly? The primary reason, among several, is that the RM2 model started with a variety of observations (transform fault positions/orientations, data about sea-floor spreading from marine magnetic anomalies, etc.) of the sort we thought about in chapter 6. These input data have uncertainties, and so each angular velocity derived from them has uncertainties. Minster and Jordan (1978, following Minster and others, 1974) then created a numerical model of angular velocity data for 11 plates (Africa, Antarctica, Arabia, Caribbean, Cocos, Eurasia, India, Nazca, North America, Pacific, South America), resulting in the 27 rotation poles reported in RM2. Their final model was an attempt to minimize uncertainties across these many plates, and the statistical errors they reported range from 0.6° to almost 17°. Subsequent models have expanded and improved the imput data, reduced uncertainties and increased the number of plates considered, but it would be difficult to overstate how important the RM2 model was in its day and what a significant achievement it was.

Exercise 8-1. You have a three plate system in which ${}_{A}\omega_{B}$ = 0.583°/Myr, ${}_{B}\omega_{C}$ = 0.937°/Myr, and ${}_{C}\omega_{A}$ = 1.248°/Myr. Write a brief *Mathematica* notebook to determine the angular distances between the three poles ${}_{\mathrm{A}}P_{\mathrm{B}}$, ${}_{\mathrm{B}}P_{\mathrm{C}}$ and ${}_{\mathrm{C}}P_{\mathrm{A}}$.

Exercise 8-2. You have been able to establish two instantaneous angular velocity vectors: ${}_{A}\Omega_{B}$ and ${}_{B}\Omega_{C}$. The three components of each of these vectors gives information about their orientation in space, and the magnitude of the vector gives information about the angular speed. Write a brief *Mathematica* notebook to determine the instantaneous velocity vector ${}_{C}\Omega_{A}$. Hint: this is a 2-dimensional triangle problem.

8.4 Instantaneous kinematics in a 2+ plate system

A definition of "2+plate system" is in order. In this scenario, we have two plates, A and B, and we can determine their instantaneous motion relative to each other: ${}_{A}\Omega_{B}$ corresponding to pole ${}_{\mathrm{A}}P_{\mathrm{B}}$. Instead of a third plate, we have a frame of reference that is external to the lithospheric plate system in which the instantaneous motion of plate A and plate B can be described: ${}_{E}\Omega_{A}$ and ${}_{E}\Omega_{B}$. The subscript "E" indicates that the frame of reference for the angular velocity is external to the plate system. A number of these external frames of reference have been used or proposed over the years, including one or more mantle hot spots, the no-net-rotation reference frame, and various satellite or sidereal solutions (GPS, Very Long Baseline Interferometry, Satellite Laser Rangefinding, Lunar Laser Rangefinding, *et cetera*). A very incomplete literature references for various external reference frames are collected at the end of this chapter for your further investigation, and the review paper by Blewitt (2009) is an excellent source of information about GPS and space-based geodetic methods. For the moment, let us simply say that an external reference frame exists relative to which we can measure the instantaneous motion of our two plates.

The governing equation in this scenario is

$${}_{A}\Omega_{B} + {}_{B}\Omega_{E} + {}_{E}\Omega_{A} = 0$$

All of the insights gleaned in the previous section apply to this system. The poles ${}_{\mathrm{A}}P_{\mathrm{B}}$, ${}_{\mathrm{E}}P_{\mathrm{A}}$ and ${}_{\mathrm{E}}P_{\mathrm{B}}$ are coplanar with the center of the spherical model Earth, and so are located on the same great circle. Their relative positions are a function of the corresponding angular speeds. We can "fill-in" a missing angular velocity vector given the other two angular velocities because this must be a closed vector circuit.

We will have much more to say about 2+plate systems in the next chapter.

8.5 Implications for finite motion?

Allan Cox (1973, p. 40-42) asserted that the two fundamental postulates of modern plate tectonics are as follows: "Postulate 1. The plates are internally rigid but are uncoupled from each other"; and "Postulate 2. The pole of relative motion between a pair of plates remains fixed relative to the two plates for long periods of time." Studies using geodetic GPS receivers have provided great insight into the limitations of Cox's first postulate, demonstrating that the western part of North America along with other plate-boundary regions worldwide are actively deforming and are not rigid (for example, see Kreemer and others, 2003). Postulate 2 might work well in an imaginary 2-plate world like the one we played with in chapter 7, but does it work in a 3-plate world?

Judge for yourself. Consider a system that seems typical of the current plate system (DeMets and others, 2010) in which the angular velocity vectors for a given 3-plate system are not colinear. That is, the angles between the poles are neither 0° nor 180°. The angular velocity vectors are pointing in three different directions. And let us further assume for the moment that the angular velocity vectors remain constant over the finite time interval that we are interested in. The angular velocity vectors are pointing in three different directions to start with, and they remain pointing in three different directions over time. Can a rigid plate move in a circular trajectory around one axis of rotation over a finite time interval, and simultaneously move around a different axis of rotation along a circular trajectory? Can plate A rotate around ${}_{\mathrm{B}}P_{\mathrm{A}}$ during the same finite time interval in which it rotates around ${}_{\mathrm{C}}P_{\mathrm{A}}$?

What's your answer?

My answer is "no." In a 3-plate system in which ${}_{A}\Omega_{B}$, ${}_{B}\Omega_{C}$ and ${}_{C}\Omega_{A}$ are non-zero and not colinear, no single plate can move in a circular arc around two separate poles of rotation simultaneously, such that all points on the plate remain the same distance from both of the poles. A board nailed to the floor at both ends cannot rotate around both nails at the same time. A paint spot on a bicycle tire cannot rotate around two different axles at the same time.

Allan Cox referred to this as the 3-plate problem of plate tectonics (Cox, 1973, p. 408; Cox and Hart, 1986, p. 255-258). His view of an approach to solve the problem was to suggest that a plate pair would move around their fixed pole "for awhile, then abruptly change direction and move along a slightly different" pole (Cox, 1973, p. 411). Indeed, Cox was aware of evidence for abrupt changes in the direction of relative plate motion contained within the marine magnetic anomalies of the Pacific seafloor (Menard and Atwater, 1968), but this really wasn't a solution for the 3-plate problem.

I met Allan Cox at the Geological Society of America annual meeting in 1986, just after he had published his book on plate tectonics/kinematics with Robert Brian Hart. Cox was working his way through the poster session at which I presented the first description of my dissertation work (Cronin, 1986) in which I provided a solution for the 3-plate problem. After reading through my poster, I believe his first words were, "Damn, I wish I had known about this before we published the book!" The person standing next to him then said, "Cheer up, Allan. There's a good excuse for a second edition." Cox died a few months later, and his kinematics book is still in print as I write this paragraph in 2012.

We will get to finite motion in future chapters.

8.9 References and relevant texts

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