

3-Point Problem:

Finding the orientation of a plane, given the locations and elevations of 3 points on that plane.

Steps to solve the 3-point problem posed in the map above.

1. Identify the highest point (B at 80 ft elevation) and the lowest point (A at 20 ft). Carefully draft a line between the highest and lowest point.
2. Take note of the elevation of the point with an intermediate elevation (C at 60 ft). If two of the points have the same elevation, skip down to step 8.

Now imagine that you are at point A at an elevation of 20 feet, and you begin walking in a straight line toward point B. As you walk up the slope, you will progressively gain elevation. Somewhere along the way, you will be at the same elevation as the point with the intermediate elevation -- point C at 60 ft elevation. So the question becomes "Where along the line from A to B is the elevation the same as at C?"

3. Subtract the lowest elevation (20 ft) from the highest elevation (80 ft): $80 - 20 = 60$ ft. Let's call this elevation difference α , where $\alpha = 60$ ft.
4. Subtract the lowest elevation (20 ft) from the intermediate elevation (C at 60 ft): $60 \text{ ft} - 20 \text{ ft} = 40$ ft. Let's call this second elevation difference β , where $\beta = 40$ ft.
5. Use a ruler to measure the map distance between the highest and lowest point. In this example, the map distance is ~ 45 mm.
6. We will find the point along line A-B that has the same elevation as point C by posing a proportion problem. The unknown value that we would like to compute is the map distance x from the lowest point (A) to the point along the A-B line that has the same elevation as the intermediate point (C). The ratio of β to α is the same as the ratio of x to the map distance from A to B.

$$\frac{\beta}{\alpha} \therefore \frac{x}{45 \text{ mm}}$$

$$\text{Rearranging to isolate the unknown value, } x = \frac{\beta \times 45 \text{ mm}}{\alpha}$$

7. Measure the distance x from A toward B and mark the point C'. This is the point along the A-B line that has the same elevation as C. Carefully draft the line between C and C'. Line C-C' is a strike line along the inclined plane that contains points A, B and C.

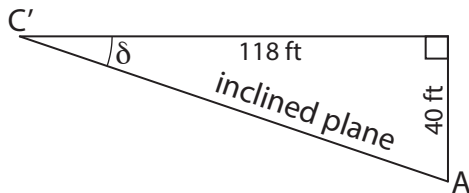
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8. Draft a line from A that is perpendicular to the C-C' line. Do the same from B to the C-C' line.
9. Measure the map length of the longer of these two lines. In this example, the line from point A to the C-C' line is longer than the line from point B, and is perhaps easier to measure because of its greater length. It is between 29 and 30 mm long, so we will estimate the map length of the line between A and C' to be 29.5 mm.
10. We need to convert the 29.5 mm map distance measured in step 9 to the actual distance at full scale as we would encounter it in the field, expressed in feet -- Imperial length units rather than metric length units. It is convenient to solve this as a proportion problem, as we did in step 6. The map length of the bar scale is 25 mm, and it represents 100 feet in the field. So 100 ft is to 25 mm as the unknown x ft is to 29.5 mm.

$$\frac{100 \text{ ft}}{25 \text{ mm}} \ddot{=} \frac{x \text{ ft}}{29.5 \text{ mm}}$$

Rearranging to isolate the unknown value, $x = \frac{100 \text{ ft} \times 29.5 \text{ mm}}{25 \text{ mm}} = 118 \text{ ft}$

11. The next step involves constructing a vertical cross section between A and C'. The slope of the inclined plane as represented in this cross section will be the true dip (δ) of the plane, because our cross section was constructed perpendicular to strike. We know that the elevation change between A and C' is 40 ft and the horizontal distance is 118 ft.



$$\tan[\delta] = \frac{40 \text{ ft}}{118 \text{ ft}} \text{ hence } \delta = \arctan\left[\frac{40 \text{ ft}}{118 \text{ ft}}\right] = \sim 19^\circ$$

The true dip angle d is 18.7° or 19° as expressed in integer degrees.

12. The dip direction is perpendicular to strike in the direction that a ball would roll down the inclined surface. In this example, the ball would roll from C' at 60 feet elevation toward A at 20 feet elevation, or toward the southwest.
13. We will express the strike azimuth using the right-hand-rule (RHR) convention. We can think of the strike line as potentially being described by either of two azimuths that are 180° apart, so the RHR rule convention directs us to choose the azimuth that is 90° counter-clockwise (or right-handed) from the dip direction. In this example, the dip direction is toward southwest, so the RHR strike direction is toward the southeast. We can measure the azimuth of line C-C' using a carefully drafted map that has a clear indication of the direction of true north. Azimuths are horizontal angles (angles measured on a horizontal plane) starting at due north and increasing clockwise to the line whose azimuth you want to measure.

In this example, the RHR strike azimuth is between 116° and 117° . Using the typical format for a RHR strike-and-dip, the orientation of the plane is "116.5, 19" meaning that it has a strike azimuth of 116.5° and a dip angle of 19° .